The Markov-Modulated Central-Place Forager: Phase Transitions and Home-Range Geometry.

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We introduce a novel discrete-time central-place foraging process in $\mathbb{R}^2$. The model is modulated by a latent, finite-state Markov chain representing patch-occupancy behaviors superimposed on environmental advection (e.g., wind or prey corridors), each with a characteristic mean drift vector μ_i. In particular, the restorative homing drive is modeled with a strictly constant intensity α, decoupling directional urgency from distance.

Our main theoretical contributions reveal a sharp, macroscale phase transition governed entirely by the ratio of the biological homing capacity α to the integrated environmental/behavioral drive $\alpha_c = \|\bar{\mu}\|$, characterizing two fundamental regimes. In the residency regime ($\alpha > \alpha_c$), the joint process is geometrically ergodic, admitting a unique invariant probability measure with exponentially bounded spatial tails. We prove a central limit theorem (CLT) for the empirical center of mass and show that the area of the trajectory's minimum convex polygon (MCP) scales as $O((\log N)^2)$ almost surely.

In the dispersal regime ($\alpha < \alpha_c$), the process becomes transient, satisfying a strong law of large numbers as the animal escapes ballistically with a deterministic velocity. The MCP area scales as $O(N^{3/2})$. In addition, we derive a fine-fluctuation CLT showing that the escaping trajectory undergoes transverse variance contraction. Directional variance projected orthogonal to the escape corridor is strictly suppressed by the analytical factor $\frac{\alpha_c - \alpha}{\alpha_c + \alpha}$, creating a self-stabilizing, narrow migratory channel.

We demonstrate the framework's versatility using a real-world ecological parameterization for pelagic seabirds navigating dynamic Southern Ocean wind fields, transitioning from range-resident to a transient dispersal phase. The results offer a unified, mathematically rigorous foundation for modeling behavioral transitions across home-range boundaries.

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